

# A Renormalization Group Analysis of the Higgs Boson with Heavy Fermions and Compositeness

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## Abstract

We study the properties of heavy fermions in the vector-like representation of the electro-weak gauge group  $SU(2)_W \times U(1)_Y$  with Yukawa couplings to the standard model Higgs boson. Using the renormalization group analysis, we discuss their effects on the vacuum stability and the triviality bound on the Higgs self-coupling, within the context of the standard model (i.e., the Higgs particle is elementary). Contrary to the low energy case where the decoupling theorem dictates their behavior, the inclusion of heavy fermions drastically changes the standard model structure at high scales. We also discuss the interesting possibility of compositeness, i.e., the Higgs particle is composed of the heavy fermions using the method of Bardeen, Hill and Lindner [1]. Finally we briefly comment on their possible role in explaining  $R_b$  and  $R_c$ .

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Enormous efforts have been made in searching for physics beyond the standard model but up to now a crucial, direct experimental indication is still illusive. One of the most important motivation to explore heavy fermions above the energy accessible by current accelerators is to look for extra building blocks of nature beyond the three families of the standard model. For this purpose it may be adequate to look at fermions in vector-like representations of the electro-weak gauge group with a large bare mass term, rather than the conventional chiral doublets. The main reason for this is from the strong experimental constraint on the S parameter [2]. While the experiments favour a negative value of S [3], a standard chiral doublet of heavy fermions (degenerate in mass) contributes to the S parameter as  $1/6\pi$ . On the contrary, for fermions in the vector-like representation of the electro-weak gauge group a large bare fermion mass  $M$  completely changes the low energy properties of the heavy fermions. As a consequence of the decoupling theorem, heavy fermions contribute to the S and T parameters as<sup>1</sup>,

$$S \sim \frac{m^2}{M^2}, \quad T \sim \frac{m'^2}{M^2} \cdot \frac{m'^2}{v^2}, \quad (1)$$

where  $m$  ( $m'$ ) is the mass parameter generated by the Yukawa coupling responsible for the electro-weak symmetry breaking (weak isospin violation),  $v$  ( $=246$  GeV) is the vacuum expectation value of the Higgs particle. The interesting aspect of the vector-like heavy fermions is not that they are safe at low energies, but most importantly, because they may have some interesting role in physics beyond the standard model. For example, they may be responsible for a dynamical generation of light fermion mass matrix [4], they are natural consequences of many grand unification models, and they may even be the constituents of the electro-weak Higgs particle [5]. Therefore it is important to investigate the fundamental properties of the heavy vector-like fermions and this is the purpose of the current paper. The model we will study in this paper is the minimal standard model plus these hypothetical heavy vector-like fermions with Yukawa interactions to the Higgs boson. In most cases bosons have just the opposite property to that of fermions. To be specific, in this paper we assume no elementary boson exists except for the known gauge bosons (and maybe the Higgs particle), in particular we assume no super-symmetry.

Recently there has been a renewed interest in understanding the structure of the standard model at high energies, even up to Planck scale [6, 7, 8]. A powerful tool is to use the renormalization group equations to trace the evolution of the coupling constant of the  $\lambda\phi^4$  self-interaction of the Higgs particle. Assuming the standard model remains valid up to certain scale  $\Lambda$ , an upper bound (the triviality bound, obtained by requiring  $\lambda$  not to blow up below  $\Lambda$ ) of the Higgs boson mass,  $m_H$ , can be obtained. Meanwhile, requiring the stability of the

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<sup>1</sup>This is also true when  $M$  is generated by chiral dynamics at higher scale but irrelevant to electro-weak physics. For the definition of the T parameter and other low energy observables, see also ref. [2].

electro-weak vacuum, we can also obtain a lower bound on  $m_h$ . For the later purpose, in principle one needs to consider the renormalization group improved effective potential [9] and require it bounded from below. But in practice this turns out to be equivalent to the requirement that the Higgs self-interaction coupling constant  $\lambda$  does not become negative, below the given scale (see [6] and ref. therein). It is remarkable to note that for the given experimental value of the top quark mass (here we use  $m_t = 174\text{GeV}$ ), there is an allowed range for the Higgs boson mass,  $130\text{GeV} \leq m_H \leq 200\text{GeV}$ , for which the standard model may remain valid up to Planck scale.

For the purpose of having a qualitative understanding of the influence of heavy fermions to the above analysis, it is enough to consider the following simple Lagrangian,

$$\mathcal{L} = \bar{Q}(\not{D}_d - M)Q + \bar{U}(\not{D}_s - M)U + \bar{D}(\not{D}_s - M)D + \{g_Y \bar{Q}\phi D + g'_Y \bar{Q}\tilde{\phi}U + h.c.\} . \quad (2)$$

In above Lagrangian we introduced four vector-like fermions,  $Q$  is a  $SU(2)_W$  doublet and  $U$  and  $D$  are singlets. We assume they participate in strong interactions and are in fundamental representations of  $SU(3)_C$ . They are equivalent to one family of chiral quarks plus a left-right conjugated chiral quark family. We call them one family of vector-like quarks. The subscript  $d$  ( $s$ ) in the covariant derivatives denotes that the corresponding fermion is a  $SU(2)_W$  doublet (singlet) and  $\phi$  denotes the standard Higgs doublet. We further expect the Yukawa couplings ( $g_Y$  and  $g'_Y$ ) to be of order 1. For simplicity we take all the bare fermion masses to be equal.

When heavy fermions are included the structure of our world changes drastically at high scales, even though vector-like fermions are essentially decoupling below their thresholds. At scales much higher than the threshold whether the fermion field is chiral or vector-like does not make any qualitative difference. The picture obtained from the following analysis also holds for Yukawa couplings between chiral quarks or between chiral and vector-like quarks.

The correction to the effective potential from the heavy fermion is

$$\begin{aligned} \delta V(\phi_c) = & -\frac{N_c}{16\pi^2} \left\{ (M + g_Y \phi_c)^4 \log\left(\frac{(M + g_Y \phi_c)^2}{\mu^2}\right) + (M - g_Y \phi_c)^4 \log\left(\frac{(M - g_Y \phi_c)^2}{\mu^2}\right) \right\} \\ & + (g_Y \rightarrow g'_Y) . \end{aligned} \quad (3)$$

As is well known, because of the negative sign, fermions turn to destabilize the vacuum. At a scale  $\phi_c < M$  one can expand the above expression in powers of  $\phi_c^2/M^2$  and it is easy to verify that heavy fermions decouple from the effective potential as a consequence of the decoupling theorem. Far above the threshold there is no difference between chiral and vector-like fermions. The only essential ingredient is the number of independent Yukawa couplings (notice that  $\phi_c \bar{Q}U$

and  $\phi_c \bar{U} Q$  are counted as different).<sup>2</sup>

In the following we list the relevant renormalization group equations (for simplicity we neglect the  $SU(2)_W \times U(1)_Y$  couplings  $g_2$  and  $g_1$ , since their effects are relatively small and do not change the qualitative picture):

$$16\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 + 12\lambda(g_t^2 + 4N_d g_Y^2) - 6(g_t^4 + 4N_d g_Y^4) , \quad (4)$$

$$16\pi^2 \frac{dg_Y}{dt} = \frac{(24N_d + 3)}{2} g_Y^3 - 8g_s^2 g_Y , \quad (5)$$

$$16\pi^2 \frac{dg_s}{dt} = -(21 - 8N_c \theta)/3 g_s^3 \quad (6)$$

where  $g_t$  is the Yukawa coupling of the top quark ( $g_t = \sqrt{2}m_t/v$ ) and  $g_s$  is the strong coupling constant. The beta function for top quark Yukawa coupling is the same as the standard model one at 1 loop we therefore do not list it here. We take  $g_Y = g'_Y$  for simplicity. The renormalization group equations are written in a more general case:  $4N_d$  is the number of independent Yukawa couplings.  $4N_c$  is the number of colored heavy fermions. For example  $N_d = 1/2$  corresponds to setting  $g_Y$  (or  $g'_Y$ ) in eq. (2) to zero. We use a simple step function  $\theta = \theta(t - \log(M/M_z))$  to model the heavy fermion threshold effects. All the  $g_Y$  couplings in above renormalization group equations are understood as multiplied by  $\theta$ .

In fig. 1 we plot the vacuum stability bound and the triviality bound on the Higgs mass as a function of the scale  $\Lambda$  for some typical values of the parameters of the heavy fermions. We see that the inclusion of heavy fermions drastically change the Standard model structure at high energies even though they decouple from the low energy world. They always tighten the bound on the mass of the Higgs boson as a function of the cutoff scale  $\Lambda$ . Notice that in principle (in terms of one loop renormalization equations) the upper line (triviality bound) and the lower line (vacuum stability bound) never meet each other. Because the upper line is drawn by requiring  $\lambda$  not to blow up and the lower line is drawn by requiring  $\lambda \geq 0$ . Between them is the ultra-violet unstable fixed point of  $\lambda$ , so the two lines get close to each other rapidly.

We now study the interesting possibility of considering the Higgs particle as a composite object of the heavy vector-like fermions. Assuming the dynamical symmetry breaking occurs through a mechanism *à la* Nambu–Jona-Lasinio (via a four-fermi interaction) or through an effective Higgs–Yukawa interaction, it is proven in [5] that, after integrating out the heavy fermion degrees of freedom,

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<sup>2</sup>There is another theoretical approach in the literature which can be related to the studies on Yukawa couplings: the Veltman condition. However in the Veltman condition there are no decoupling or threshold effects. The condition simply counts the number of independent Yukawa couplings (and of course, their strength). In this paper, we do not investigate this approach.

the model is completely equivalent to the standard model at low energies and the Higgs boson's mass has nothing to do with the bare fermion mass, even though the Higgs boson is composed of heavy fermions. Applying the above renormalization group analysis to the composite model leads to some interesting results which we present below. We follow the method proposed by Bardeen, Hill and Lindner (BHL) [1] originally developed for the top quark condensate model. The basic idea of the BHL method is the following: Using the collective field method the four-fermi interaction Lagrangian is rewritten into an effective Higgs-Yukawa interaction Lagrangian at the cutoff scale  $\Lambda$  (the coupling strength of the four-fermi interaction is  $\sim G/\Lambda^2$ , where  $G$  is a dimensionless coupling constant). The effective Yukawa interaction Lagrangian is identical to the standard model at the cutoff scale  $\Lambda$ , but with vanishing wave function renormalization constant of the Higgs field ( $Z_H = 0$ ) and vanishing Higgs self-coupling ( $\lambda = 0$ ). Below  $\Lambda$  the model is equivalent to the standard model and therefore the coupling constants of the effective theory run according to the standard model renormalization group equations. However the vanishing of  $Z_H$  at the scale  $\mu = \Lambda$  leads to the following boundary conditions of the renormalization group equations:

$$g_Y^r \rightarrow \infty, \quad \lambda^r/(g_Y^r)^4 \rightarrow 0 \quad (7)$$

where  $\lambda^r$  and  $g_Y^r$  are the renormalized Higgs self-coupling and Yukawa coupling, respectively. With the renormalization group equations and boundary conditions, one can predict the mass of the Higgs boson and the fermion mass (or the Yukawa coupling) at its infra-red fixed point value. In the present paper, of course, the “standard model” often refers to the standard model plus heavy fermions and the “infra-red fixed point” value of  $g_Y$  refers to its value at the heavy fermion threshold.

The minimal top quark condensate model has already been ruled out by experiments. In order to generate the electroweak symmetry breaking scale  $v$ , the top quark mass is required to be at least as large as 218 GeV (corresponding to  $\Lambda = 10^{19}$  GeV, i.e., Planck scale). The experimental bound on the top quark mass ( $180 \pm 13$  GeV [10]) indicates that the top quark Yukawa coupling does not diverge up to Planck scale in the standard model and therefore does not meet the compositeness condition of BHL (see however [11]). Since in the present model there is no strict experimental constraint, the compositeness condition is easily and naturally achievable. In fig. 2 we plot the composite Higgs particle's mass<sup>3</sup> and the Yukawa coupling  $g_Y$  at the infra-red fixed point, as a function of  $\Lambda$ . For a large number of colored fermions there is a problem with non-asymptotic freedom of the strong gauge coupling. Non-asymptotic freedom already happens with 3 vector-like quark families. However for  $N_c = 3$  the strong coupling constant behaves mildly up to Planck scale and causes no problem. For simplicity we set all the Yukawa couplings to be equal.

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<sup>3</sup>The Higgs mass in these figures is the renormalized mass at  $\mu = M_Z$ . The renormalized mass is close to the pole mass of the Higgs boson.

From fig. 2 we see that the allowed range for the Higgs mass is rather narrow against the wide range of the cutoff scale, the bare fermion mass and the number of families except when the heavy fermion bare mass  $M$  is close to the cutoff  $\Lambda$ . A lower bound on the Higgs mass can be obtained:  $m_H \geq 145$  GeV. When  $M$  is getting close to the cutoff scale our results become unstable and are sensitive to the input numerical values of the boundary conditions. In such a situation the scale is not large enough for the couplings to reach the infra-red stable point. However it is estimated that the Higgs mass will not exceed 450 GeV, otherwise the whole mechanism become unnatural (in the sense that the Yukawa coupling constant at electroweak scale also becomes substantially larger than 1).

In fig. 3 we plot a typical example of the Higgs mass for a given cutoff scale  $\Lambda_c$  and  $N_d$  ( $N_c$ ). We also plot the triviality bound and the vacuum stability bound using the Yukawa coupling constant at its infrared fixed-point value as the initial boundary condition, which is determined uniquely by the parameter  $M$ ,  $\Lambda_c$  and  $N_d$  ( $N_c$ ) in the compositeness picture. It is very interesting to notice that  $m_H$  and  $\Lambda$  take the values where the curves of triviality bound and vacuum stability bound (practically) meet each other. This is the unique feature of BHL compositeness picture. The reason behind this is very simple: The infra-red attractive fixed point corresponds to the ultra-violet unstable fixed point. In the sense of ref. [11], this picture can be disturbed. However in most cases the infra-red-ultra-violet fixed point structure is influential and rather stable against perturbation. From the above analysis, it is clear that the Higgs particle's mass obtained in a composite model will not contradict <sup>4</sup> the triviality bound obtained from its low energy theory. Without a renormalization group analysis, the obtained Higgs mass in the limit of large number of colors like the relation  $m_H = 2m_t$  in top condensation model or that  $m_H$  equals to the mass splitting between heavy vector-like fermions in the current model does often contradict the triviality bound<sup>5</sup>.

Recently several authors [13] have suggested that a proper mixing between heavy vector-like fermions and the  $b, c$  quarks may resolve the  $R_b$  ( $\equiv \Gamma_b/\Gamma_{hadron}$ ) and  $R_c$  ( $\equiv \Gamma_c/\Gamma_{hadron}$ ) crisis. In the second model of Chang, Chang and Keung's paper in ref. [13], one weak singlet, charge 2/3 heavy quark which mixes with the charm quark is introduced to reduce  $R_c$ , and a vectorial  $SU(2)_W$  doublet with weak hypercharge  $-5/3$  mixing with the down-type quarks is introduced to increase  $R_b$ . The influence on  $R_c$  and  $R_b$  depends on the mixing angle between the vectorial and the chiral quarks. The mixing angle,  $\theta \sim g_Y v/\sqrt{2}M$  can be determined from the experimental value of  $R_b$  and  $R_c$ . For sufficiently large  $M$ , for example,  $M \sim 1$  TeV <sup>6</sup>, a rough estimate indicates that the Yukawa

<sup>4</sup>That is, the low energy model should not blow up before reaching the compositeness scale.

<sup>5</sup>For the discussion on the relations between the Higgs boson mass obtained from the renormalization group analysis and that from a large  $N_c$  analysis, see [12].

<sup>6</sup> $M$  cannot be taken too large in order to be consistent with the experimental constraints on the T parameter. Fixed mixing angle requires the increasing of the Yukawa coupling when

coupling is already large enough to meet the compositeness condition (i.e., the running Yukawa coupling blows up below the Planck scale.). However a more quantitative analysis requires considering also the running of  $g_2$  and  $g_1$  couplings.

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$M$  is increased.

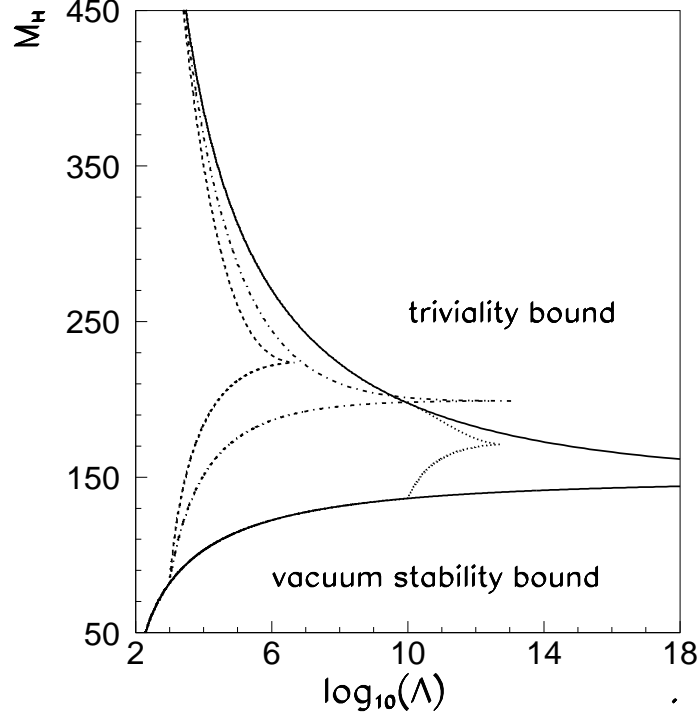


Figure 1: Vacuum stability and triviality bounds on the Higgs mass as a function of  $\Lambda$ . The solid lines are the standard model case (in the absence of  $g_2$  and  $g_1$ , we take  $\alpha_s = 0.118$ ); The dot-dashed lines correspond to  $N_d = 1/2$ ,  $N_c = 3/4$  (i.e., without “ $U$ ” or “ $D$ ” quark in eq. (2)) and  $M = 10^3$  GeV; The dashed (dotted) lines correspond to  $N_d = 1$ ,  $N_c = 1$  and  $M = 10^3$  GeV ( $M = 10^{10}$  GeV), respectively. The Yukawa coupling  $g_Y = 1$ .



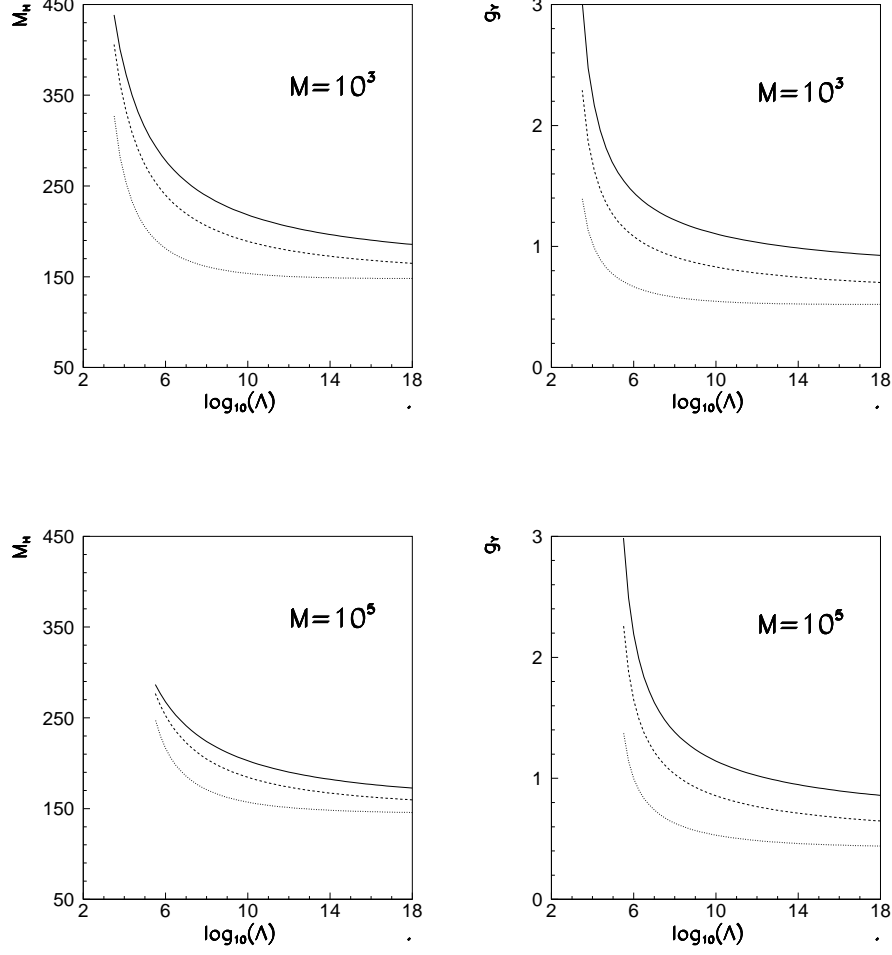


Figure 2: IR fixed point value (at  $M_Z$ ) of  $M_H$  and  $g_Y$  as functions of the compositeness scale. The solid line:  $N_d = 1/2, N_c = 3/4$ ; the dashed line:  $N_d = N_c = 1$ ; the dotted line:  $N_d = N_c = 3$ . The value of  $M$  in the figure is in unit of GeV.

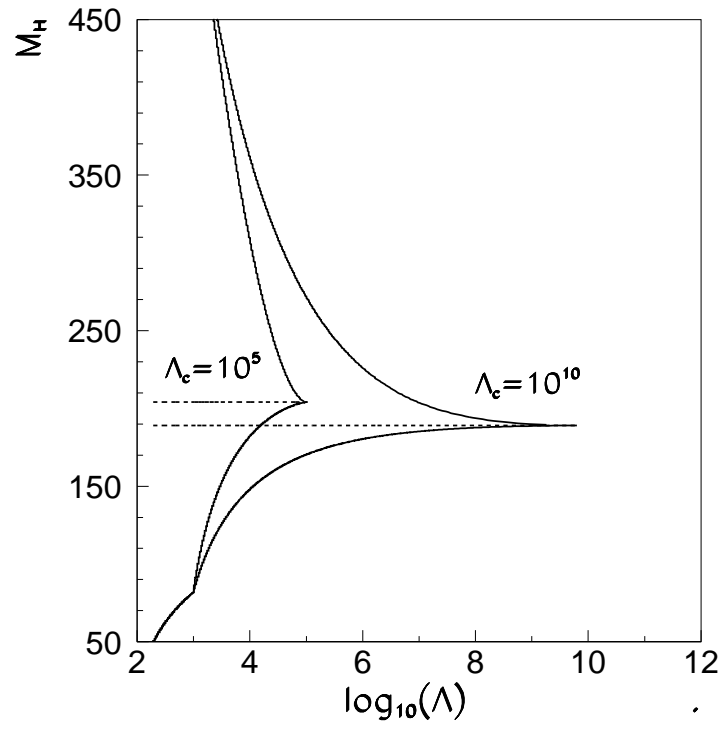


Figure 3: IR–UV fixed point structure and compositeness. In the  $\Lambda_c = 10^5$  GeV case,  $N_d = N_c = 3$ ; In the  $\Lambda_c = 10^{10}$  GeV case,  $N_d = N_c = 1$ .  $M = 10^3$  GeV.